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Modeling The Carbon Cycle With A Box-Model

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1 Introduction

This project aims to create a model that will predict the global temperature changes from current time until the year 2100. The model is based on the box model for the carbon cycle, which was created by Akira Tomizuka Tomizuka (2009).

The model starts by predicting the future concentration of CO_2 in the atmosphere based on various hypothetical scenarios for the amount of CO_2 emissions that are released to the environment, then it proceeds to use the concentrations of CO_2 in the atmosphere to predict the future temperature change for each scenario.

With this approach, it is possible to to predict the change of global temperature based on different levels of CO_2 emissions. The result of this project give insights on the affects of human activities on the environment and highlights the importance of taking preventative actions to mitigate the danger of climate change.

2 Theoretical background

This section contains an overview of theoretical backgrounds that were used in the making of this paper.

The carbon box model

According to the the paper by Tomizuka Tomizuka (2009). The box model consists of a system of seven differential equations (Eq 1 to Eq 7). Those differential equations represent rate-equations for the the following compartments:

1. Atmosphere
2. Surface ocean
3. Intermediate ocean
4. Deep ocean
5. Sediments
6. Biosphere
7. Soil

$$\frac{dN_1}{dt} = -k_{12}N_1 + k_{21}(N_2^0 + \xi(N_2 - N_2^0)) + \gamma - f + \delta + k_{51}N_5 + k_{71}N_7 \tag{1}$$
$$\frac{dN_2}{dt} = k_{12}N_1 - k_{21}(N_2^0 + \xi(N_2 - N_2^0)) - k_{23}N_2 + k_{32}N_3 - k_{24}N_2 \tag{2}$$
$$\frac{dN_3}{dt} = k_{23}N_2 - k_{32}N_3 - k_{34}N_3 + k_{43}N_4 \tag{3}$$
$$\frac{dN_4}{dt} = k_{34}N_3 - k_{43}N_4 + k_{24}N_2 - k_{45}N_4 \tag{4}$$
$$\frac{dN_5}{dt} = k_{45}N_4 - k_{51}N_5 \tag{5}$$
$$\frac{dN_6}{dt} = f - k_{67}N_6 - 2\delta \tag{6}$$
$$\frac{dN_7}{dt} = k_{67}N_6 - k_{71}N_7 + \delta \tag{7}$$

The term k_{ij} represents the transfer coefficient from compartment i to compartment j. The preindustrial transfer coefficients have the values in table 1:

$k_{12} = 60/615$	$k_{21} = 60/842$	$k_{23} = 9/842$
$k_{24} = 43/842$	$k_{32} = 52/9744$	$k_{34} = 162/9744$
$k_{43} = 205/26280$	$k_{45} = 0.2/26280$	$k_{51} = 0.2/90000000$
$k_{67} = 62/731$	$k_{71} = 62/1238$	

Table 1: The preindustrial transfer coefficients Tomizuka (2009)

The term ξ is a function that depends on atmospheric CO_2 concentration P and can be calculated according to Eq 8

$$\xi(P) = 3.69 + 1.86 \times 10^{-2}P - 1.80 \times 10^{-6}P^2 \tag{8}$$

N_2^0 is the preindustrial content of carbon in the sea surface and has the value 842 PgC
The term γ is the carbon emission in PgC due to fossil fuel burning.
The term δ is the carbon emission in PgC due to land usage.
The term f is the net CO_2 flux to the biosphere, and is calculated according to Eq 9

$$f = f_0\left(1 + \beta \ln\left(\frac{P}{P_0}\right)\right) \tag{9}$$

Where:

- f_0 = 62 PgC/year is the preindustrial flux,
- P_0: preindustrial atmospheric concentrations of CO_2 (289 ppm)
- P: current atmospheric concentrations of CO_2
- β: the fertilization factor. (Default value $\beta = 0.42$)

Solving the box model gives N_1 from the first compartment, which is the content of carbon in the atmosphere for each year (To get the concentration of CO_2 we divide by 2.13).

Runge-kutta methods

Runge-Kutta methods is a family of numerical methods that are used to solve first order differential equations, Runge-Kutta 4 is the most used algorithm in the family, and is presented bellow.

Given a first order differential equation $y' = f(t, y)$ and the initial values t_0, y_0. We can use Runge-kutta 4 to solve the equation in the range $[a, b]$ as follows:

1. Choose N which is the number of points to solve for (more points generally lead to more accurate results.).
2. Generate a vector t with all the numbers to solve for, this is done by generating N evenly spaced numbers within the range $[a, b]$. (In python numpy, this can be done with np.linspace.)
3. Calculate the step size h using the equation: $h = \frac{b-a}{N}$
4. For each number in t we can calculate the solution as follows:
   - We define k_1, k_2, k_3, k_4 as follows:
     $k_1 = h \times f(t_n, y_n)$
     $k_2 = h \times f(t_n + h/2, y_n + k_1/2)$
     $k_3 = h \times f(t_n + h/2, y_n + k_2/2)$
     $k_4 = h \times f(t_n + h, y_n + k_3)$
     Where:
     - k_1: the derivative (slope) at the beginning of the interval.
     - k_2: the derivative (slope) at the midpoint of the interval (using y and k_1).
     - k_3: the derivative (slope) at the midpoint of the interval (using y and k_2).
     - k_4: the derivative (slope) at the end of the interval.

- We calculate the next value of the solution y at time t_{n+1} using the weighted average of k_1, k_2, k_3, k_4:
  $y_{n+1} = y_n + 1/6(k_1 + 2k_2 + 2k_3 + k_4)$
  Note: k_2 and k_3 have double the accuracy of k_1 and k_4, thus we multiply them with 2.

It is worthy to note that Runge-Kutta 4 can be expanded to solve systems of differential equations, this can for example be implemented in code by treating the system of differential equations as a single function that returns the solution to each of the differential equations. Moreover, because higher order differential equations can be converted to first order differential equations, Runge-Kutta 4 can be used to solve higher order differential equations and systems of higher order differential equations.

3 Method

To be able to predict the future global temperature we need to calculate the the concentration of CO_2 in the atmosphere. And then use the concentration of CO_2 to predict the future global temperature with the help of the theory of climate sensitivity.
The concentration of CO_2 will be calculated using the box model for the carbon cycle, which was created by Akira Tomizuka Tomizuka (2009)

This project uses the following steps to reach its results:

Step 1: getting the data

To be able to solve the box model we need to have the historical data of CO_2 emission from the burning of fossil fuel and from land usage. For this paper we use the emission dataset provided in Ritchie & Roser (2020). The dataset contains lots of data that is not required for this paper, so we have extracted only the data from the rows "world". Moreover the dataset has the emissions in unit of million tonnes, but they have been converted to PgC, since that is a requirement for the box model.
The historical emission data is visualized in fig 1

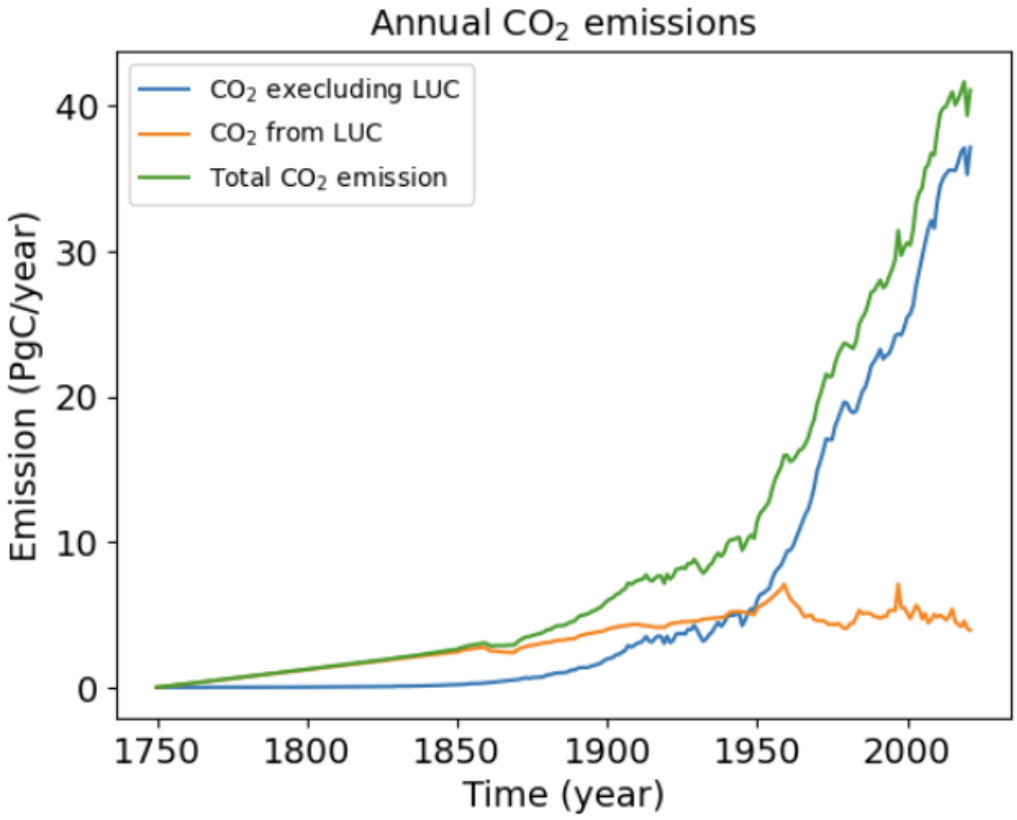

Figure 1: Historical emission data

Step 2: Solving the box model for the years 1750 to 2021(current time)

Before we can use the box model to predict future CO_2 concentration, we need to make sure that the box model works properly, moreover we may need to fine tune the free parameters

of the model (in this case β). Thus we solve the system from the year 1750 to 2021 first.
Note: the parameter β is a free parameter that was tuned by testing it with different values till the concentration of CO_2 given by the model matches the current concentration. The value of β has to be around 0.42 Tomizuka (2009). The concentration of CO_2 should be 414 ppm in 2021 Lindsey (2022), and after experimenting with different values for β we found that the value $\beta = 0.568$ gives the concentration 414 ppm.
This paper uses numerical methods to solve the system of differential equations. Specifically the paper uses Runge-Kutta method, with the exact implementation being the one in `"scipy.integrate.solve_ivp"` SciPy-Community (n.d.).
Solving the box model gives the concentration of CO_2 in atmosphere, which is visualized in fig 2.

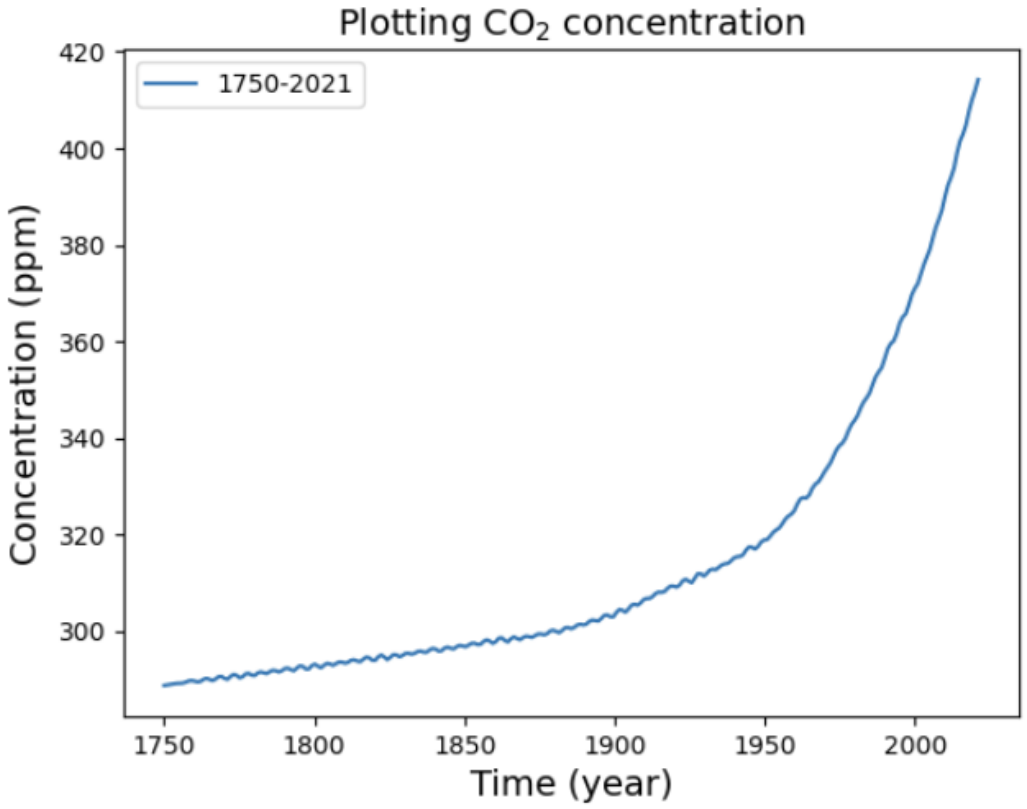

Figure 2: Concentration 1750-2021

Step 4: creating future emission scenarios, and use the box model to calculate the future CO_2 concentration.

Now that we have created a working Carbon box model, we can start predicting future CO_2 concentrations. However to do so, we need to generate emission data for fossil fuel burning and land usage.
To generate the future emission data, the paper takes inspiration from the five scenarios presented in the IPCC report IPCC (2022*a*) Those scenarios are for CO_2 emissions till the year 2100. The scenarios can be seen in figure 3.

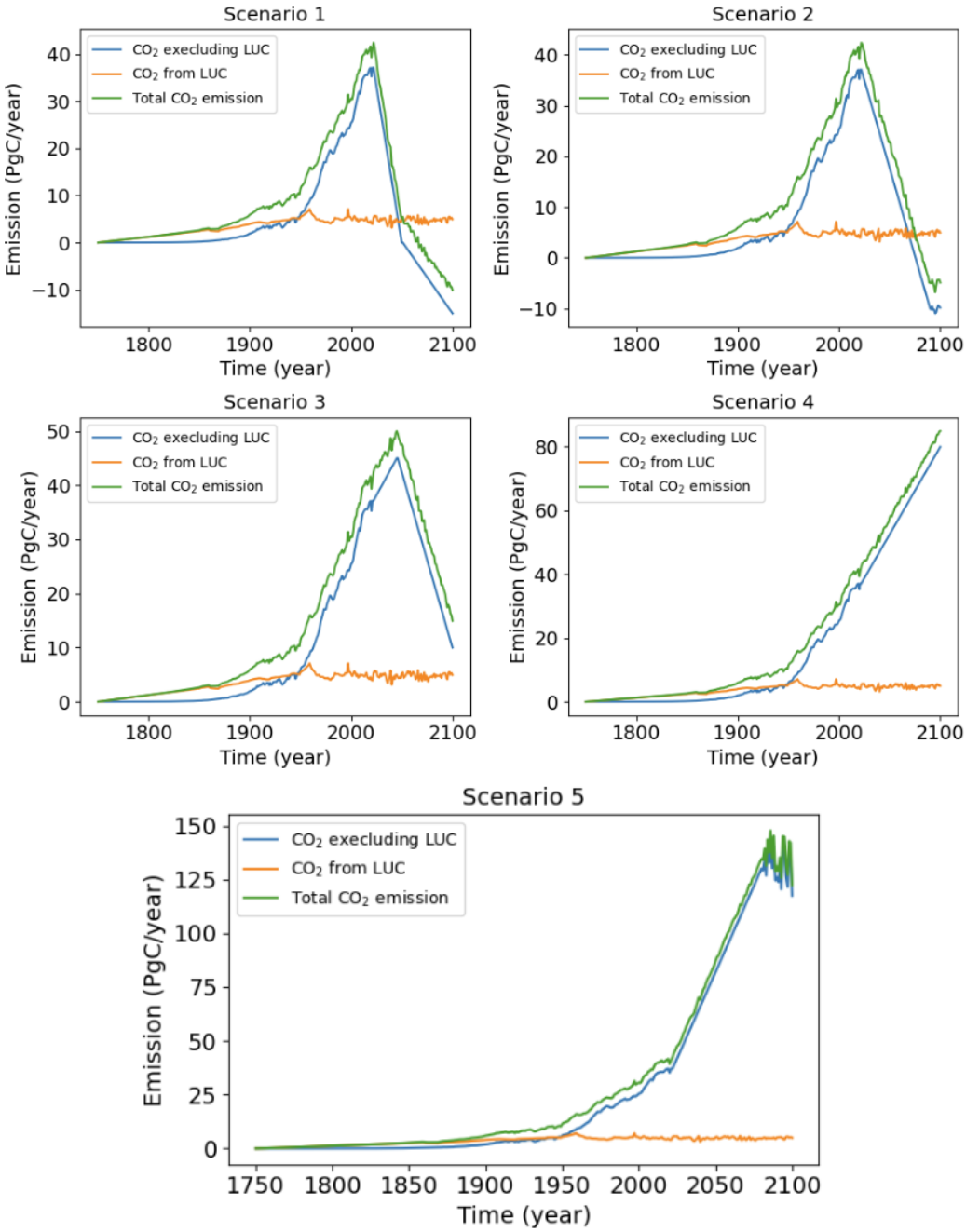

Figure 3: Emission Scenarios

After having created the emission scenarios, it is possible to solve for future CO_2 concentration using the box model, in the same way as for the years 1750-2021.

Step 5: predicting the change in global temperature based on the CO_2 concentration.

To predict the change in global temperature we can use theory of climate sensitivity. First we need to calculate the radioactive forcing using the equation 10.

$$\Delta F = 5.35 \times \ln\left(\frac{P}{P_0}\right) \tag{10}$$

Now it is possible to calculate the increase of temperature ΔT by multiplying the radioactive forcing with the climate sensitivity parameter λ as seen in the equation 11.

$$\Delta T = \lambda \Delta F \tag{11}$$

Without feedback mechanisms $\lambda = 0.27$ Km^2/W, however, feedback mechanisms are important to get accurate results, thus the value of λ was found by experiments. We know that the temperature has increased by 1.1° between the years 1850-2021 IPCC (2022*b*), thus we have experimented with different values of λ till we got a temperature increase of 1.1 degree using the value $\lambda = 0.62$.

4 Results

Solving the carbon box model for each of the scenarios gives the concentration of CO_2 till the year 2100, the results are visualized in fig 4.

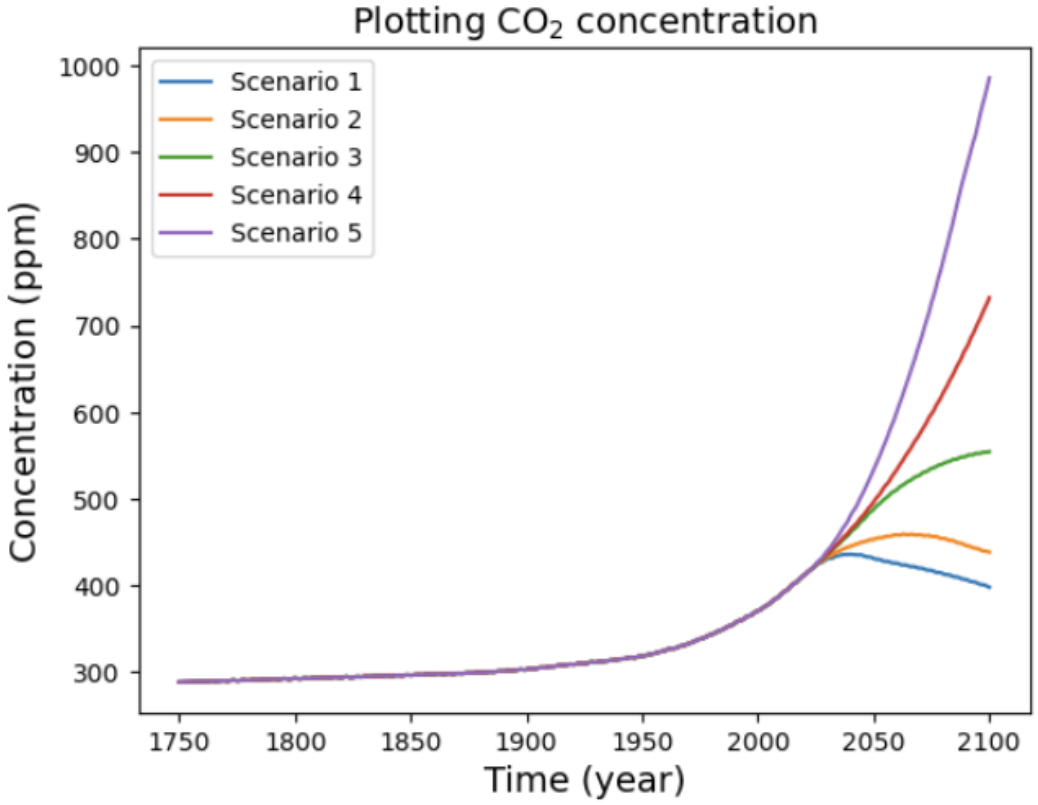

Figure 4: Concentration 1750-2100

To get the change of temperature we have used the theory of climate sensitivity as described earlier, the results for the different scenarios can be seen in fig 5.

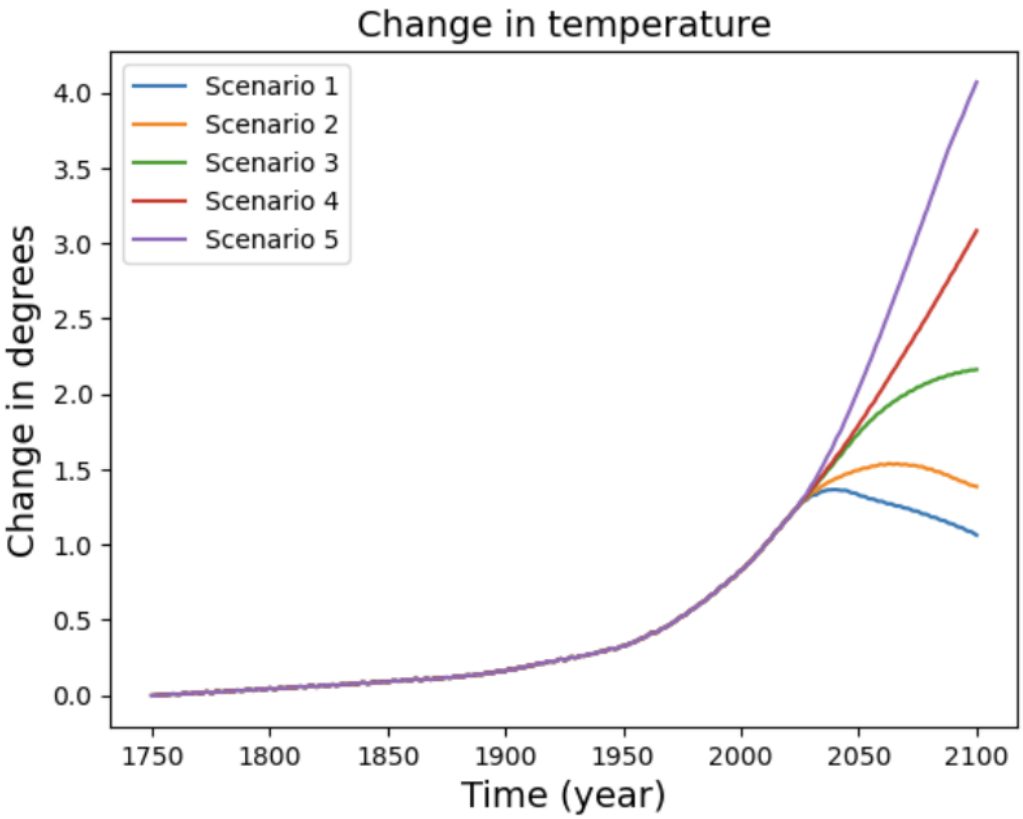

Figure 5: Temperature change

In table 2, we can see a comparison regarding the temperature change results, between this project and the IPCC report IPCC (2022*a*). The table shows the predictions for temperature change for the year 2100.
This project does not take into account emission data from green house gases other than CO_2 (for example methane and nitrous oxide), however we have tuned the parameters β and λ as explained in the method section to match the observed total temperature change, thus this project indirectly takes into account the affects of other green house gases. This means that it still makes sense to compare the result of this project with the total temperature change presented in the IPCC report.

Scenarios	**IPCC (best estimate)**	**IPCC (very likely range)**	**This project**
Scenario 1	1.4°	1.0° to 1.8°	1.0°
Scenario 2	1.8°	1.3° to 2.4°	1.3°
Scenario 3	2.7°	2.1° to 3.5°	2.1°
Scenario 4	3.6°	2.8° to 4.6°	3.0°
Scenario 5	4.4°	3.3° to 5.7°	4.0°

Table 2: Comparing the temperature change result to the IPCC report

5 Discussion

The result for the temperature change predicted by this project is in line with the "very likely range" result from the IPCC report IPCC (2022*a*), however the result falls within the lower end of the IPCC's "very likely range". The difference in results in table 2 between this project and the IPCC's best estimate are likely to be related to the emission scenarios rather than the carbon box model. This is because this project uses similar emission data, but not identical to that used by the IPCC, moreover the IPCC report considers future emissions from green house gases other than CO_2, while this project only consider future emissions of CO_2.

From the results in fig 4 we see that the increase of CO_2 emissions can quickly raise the atmospheric CO_2 concentration, however a decrease in CO_2 emissions does not quickly lower the concentration of CO_2. This is due to the slow process by which natural carbon sinks, such as oceans and forests, are able to remove the CO_2 from the atmosphere Moseman (2022).

The result of the study is consistent with previous research regarding the following two criteria:

- Indicating a strong correlation between the CO_2 emission and global temperature change.
- Showing the slow rate at witch carbon sinks can remove CO_2 from the atmosphere

The results of the project has important implications for climate policy. This is because any policies aimed at reducing CO_2 emissions have to be placed sufficiently in advance to calculate for the delay that is needed for these policies to take affect at reducing global temperature.

References

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